AP® CALCULUS AB

AP

About the Advanced Placement Program® (AP®)

The Advanced Placement Program® has enabled millions of students to take college-level courses and earn college credit, advanced placement, or both, while still in high school. AP Exams are given each year in May. Students who earn a qualifying score on an AP Exam are typically eligible to receive college credit, placement into advanced courses, or both. Every aspect of AP course and exam development is the result of collaboration between AP teachers and college faculty. They work together to develop AP courses and exams, set scoring standards, and score the exams. College faculty review every AP teacher's course syllabus.

AP Calculus Program

AP Calculus AB and AP Calculus BC focus on students' understanding of calculus concepts and provide experience with methods and applications. Although computational competence is an important outcome, the main emphasis is on a multirepresentational approach to calculus, with concepts, results, and problems being expressed graphically, numerically, analytically, and verbally. The connections among these representations are important.

Teachers and students should regularly use technology to reinforce relationships among functions, to confirm written work, to implement experimentation, and to assist in interpreting results. Through the use of the unifying themes of calculus (e.g., derivatives, integrals, limits, approximation, and applications and modeling) the courses become cohesive rather than a collection of unrelated topics.

AP Calculus AB Course Overview

AP Calculus AB is roughly equivalent to a first semester college calculus course devoted to topics in differential and integral calculus. The AP course covers topics in these areas, including concepts and skills of limits, derivatives, definite integrals, and the Fundamental Theorem of Calculus. The course teaches students to approach calculus concepts and problems when they are represented graphically, numerically, analytically, and verbally, and to make connections amongst these representations.

Students learn how to use technology to help solve problems, experiment, interpret results, and support conclusions.

RECOMMENDED PREREQUISITES

Before studying calculus, all students should complete the equivalent of four years of secondary mathematics designed for college-bound students: courses which should prepare them with a strong foundation in reasoning with algebraic symbols and working with algebraic structures. Prospective calculus students should take courses in which they study algebra, geometry, trigonometry, analytic geometry, and elementary functions. These functions include linear, polynomial, rational, exponential, logarithmic, trigonometric, inverse trigonometric, and piecewise-defined functions. In particular, before studying calculus, students must be familiar with the properties of functions, the composition of functions, the algebra of functions, and the graphs of functions. Students must also understand the language of functions (domain and range, odd and even, periodic, symmetry, zeros, intercepts, and descriptors such as increasing and decreasing). Students should also know how the sine and cosine functions are defined from the unit circle and know the values of the trigonometric functions at the numbers 0, $\pi/6$, $\pi/4$, $\pi/3$, $\pi/2$, and their multiples. Students who take AP Calculus BC should have basic familiarity with sequences and series, as well as some exposure to polar equations.

Use of Graphing Calculators

Professional mathematics organizations have strongly endorsed the use of calculators in mathematics instruction and testing. The use of a graphing calculator in AP Calculus AB is considered an integral part of the course.

The Big Ideas of AP Calculus

The course is organized around the foundational concepts of calculus:

I. Limits:
Students must have a solid, intuitive understanding of limits and be able to compute one-sided limits, limits at infinity, the limit of a sequence, and infinite limits. They should be able to apply limits to understand the behavior of a function near a point and understand how limits are used to determine continuity.

II. Derivatives:
Students should be able to use different definitions of the derivative, estimate derivatives from tables and graphs, and apply various derivative rules and properties. Students should also be able to solve separable differential equations, understand and be able to apply the Mean Value Theorem, and be familiar with a variety of real-world applications, including related rates, optimization, and growth and decay models.

III. Integrals and the Fundamental Theorem of Calculus:
Students should be familiar with basic techniques of integration, including basic antiderivatives and substitution, and properties of integrals. Students should also understand area, volume, and motion applications of integrals, as well as the use of the definite integral as an accumulation function. It is critical that students understand the relationship between integration and differentiation as expressed in the Fundamental Theorem of Calculus.

Mathematical Practices for AP Calculus

The Mathematical Practices for AP Calculus (MPACs) capture important aspects of the work that mathematicians engage in, at the level of competence expected of AP Calculus students. These MPACs are highly interrelated tools that should be used frequently and in diverse contexts to support conceptual understanding of calculus.

1. Reasoning with definitions and theorems
2. Connecting concepts
3. Implementing algebraic/computational processes
4. Connecting multiple representations
5. Building notational fluency
6. Communicating

Review Exercises

Finding Intercepts   In Exercises 1–4, find any intercepts algebraically. Use a graphing utility to verify your results.

1. $y = 5x - 8$
2. $y = x^2 - 8x + 12$ 1–4. See margin.
3. $y = \dfrac{x-3}{x-4}$
4. $y = (x - 3)\sqrt{x + 4}$

Testing for Symmetry   In Exercises 5–8, test for symmetry with respect to each axis and to the origin.

5. $y = x^2 + 4x$
6. $y = x^4 - x^2 + 3$ 5–8. See margin.
7. $y^2 = x^2 - 5$
8. $xy = -2$

Using Intercepts and Symmetry to Sketch a Graph   In Exercises 9–14, sketch the graph of the equation. Identify any intercepts and test for symmetry.   9–14. See margin.

9. $y = -\frac{1}{2}x + 3$
10. $y = -x^2 + 4$
11. $y = x^3 - 4x$
12. $y^2 = 9 - x$
13. $y = 2\sqrt{4 - x}$
14. $y = |x - 4| - 4$

Finding Points of Intersection   In Exercises 15 and 16, find the points of intersection of the graphs of the equations. Use a graphing utility to verify your results.

15. $x - y = -5$
   $x^2 - y = 1$
   $(-2, 3), (3, 8)$
16. $x^2 + y^2 = 1$
   $-x + y = 1$
   $(0, 1), (-1, 0)$

17. **Stress Test**   A machine part was tested by bending it x centimeters 10 times per minute until the time y (in hours) of failure. The results are recorded in the table.   See margin.

x	3	6	9	12	15	18	21	24	27	30
y	61	56	53	55	48	35	36	33	44	23

(a) Use the regressions capabilities of a graphing utility to find a linear model for the data.

(b) Use a graphing utility to plot the data and graph the model.

(c) Use the graph to determine whether there may have been an error made in conducting one of the tests or in recording the results. If so, eliminate the erroneous point and find the model for the remaining data.

18. **Median Income**   The data in the table show the median income y (in thousands of dollars) for females of various ages x in the United States in 2013. (*Source: U.S. Census Bureau*)   See margin.

x	20	30	40	50	60	70
y	10.2	26.4	30.6	31.0	26.9	18.0

(a) Use the regression capabilities of a graphing utility to find a quadratic model for the data.

(b) Use a graphing utility to plot the data and graph the model.

(c) Use the model to approximate the median incomes for a female who is 27 years old and a female who is 56 years old.

Finding the Slope of a Line   In Exercises 19 and 20, plot the points and find the slope of the line passing through them.   19–20. See margin.

19. $\left(\frac{3}{2}, 1\right), \left(5, \frac{5}{2}\right)$
20. $(-7, 8), (-1, 8)$

Finding an Equation of a Line   In Exercises 21–24, find an equation of the line that passes through the point and has the indicated slope. Then sketch the line.   21–24. See margin.

	Point	Slope		Point	Slope
21.	$(3, -5)$	$m = \frac{7}{4}$	22.	$(-8, 1)$	m is undefined.
23.	$(-3, 0)$	$m = -\frac{2}{3}$	24.	$(5, 4)$	$m = 0$

Sketching Lines in the Plane   In Exercises 25–28, use the slope and y-intercept to sketch a graph of the equation.   25–28. See margin.

25. $y = 6$
26. $x = -3$
27. $y = 4x - 2$
28. $3x + 2y = 12$

Finding an Equation of a Line   In Exercises 29 and 30, find an equation of the line that passes through the points. Then sketch the line.   29–30. See margin.

29. $(0, 0), (8, 2)$
30. $(-5, 5), (10, -1)$

31. **Finding Equations of Lines**   Find equations of the lines passing through $(-3, 5)$ and having the following characteristics.

(a) Slope of $\frac{7}{16}$ $y = \frac{7}{16}x + \frac{101}{16}$
(b) Slope of $-\frac{2}{3}$ $y = -\frac{2}{3}x + 3$
(c) Parallel to the line $5x - 3y = 3$ $5x - 3y + 30 = $
(d) Perpendicular to the line $3x + 4y = 8$ $4x - 3y + $
(e) Parallel to the y-axis $x = -3$
(f) Parallel to the x-axis $y = 5$

32. **Break-Even Analysis**   A contractor purchases a piece of equipment for $36,500 that costs an average of $9.25 per hour for fuel and maintenance. The equipment operator is paid $13.50 per hour, and customers are charged $30 per hour.

(a) Write an equation for the cost C of operating this equipment for t hours. $C = 22.75t + 36,500$
(b) Write an equation for the revenue R derived from t hours of use. $R = 30t$
(c) Find the break-even point for this equipment by finding the time at which $R = C$.   5034.48 hours

Summer HW 2018-2019

AP Calculus AB #1-70 Odds (show all work)

AP® CALCULUS BC

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AP Calculus BC Course Overview

AP Calculus BC is roughly equivalent to both first and second semester college calculus courses. It extends the content learned in AB to different types of equations (polar, parametric, vector-valued) and new topics (such as Euler's method, integration by parts, partial fraction decomposition, and improper integrals), and introduces the topic of sequences and series. The AP course covers topics in differential and integral calculus, including concepts and skills of limits, derivatives, definite integrals, the Fundamental Theorem of Calculus, and series. The course teaches students to approach calculus concepts and problems when they are represented graphically, numerically, analytically, and verbally, and to make connections amongst these representations.

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Use of Graphing Calculators

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AP Calculus BC Course Content

The course is organized around the foundational concepts of calculus:

I. Limits:
Students must have a solid, intuitive understanding of limits and be able to compute one-sided limits, limits at infinity, the limit of a sequence, and infinite limits. They should be able to apply limits to understand the behavior of a function near a point and understand how limits are used to determine continuity.

II. Derivatives:
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III. Integrals and the Fundamental Theorem of Calculus:
Students should be familiar with basic techniques of integration, including basic antiderivatives and substitution, and properties of integrals. Students should also understand area, volume, and motion applications of integrals, as well as the use of the definite integral as an accumulation function. It is critical that students understand the relationship between integration and differentiation as expressed in the Fundamental Theorem of Calculus.

IV. Series:
Students should be familiar with various methods for determining convergence and divergence of a series, Maclaurin series for common functions, general Taylor series representations, radius and interval of convergence, and operations on power series. The technique of using power series to approximate an arbitrary function near a specific value allows for an important connection back to the tangent-line problem.
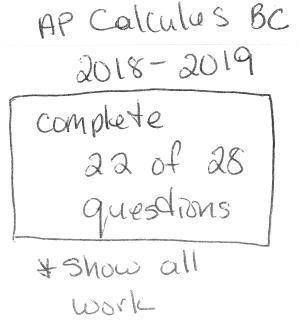
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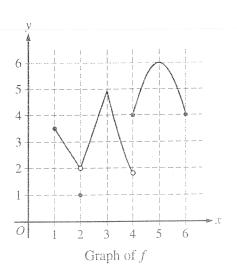5. Building notational fluency
6. Communicating

Summer Homework

A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A

1. If $y = x \sin x$, then $\dfrac{dy}{dx} =$

complete
22 of 28
questions

#Show all
work

(A) $\sin x + \cos x$

(B) $\sin x + x \cos x$

(C) $\sin x - x \cos x$

(D) $x(\sin x + \cos x)$

(E) $x(\sin x - \cos x)$

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2. Let f be the function given by $f(x) = 300x - x^3$. On which of the following intervals is the function f increasing?

(A) $(-\infty, -10]$ and $[10, \infty)$

(B) $[-10, 10]$

(C) $[0, 10]$ only

(D) $[0, 10\sqrt{3}]$ only

(E) $[0, \infty)$

Graph of f

5. The graph of the function f is shown above. Which of the following statements is false?

(A) $\lim_{x \to 2} f(x)$ exists.

(B) $\lim_{x \to 3} f(x)$ exists.

(C) $\lim_{x \to 4} f(x)$ exists.

(D) $\lim_{x \to 5} f(x)$ exists.

(E) The function f is continuous at $x = 3$.

6. A particle moves along the x-axis. The velocity of the particle at time t is $6t - t^2$. What is the total distance traveled by the particle from time $t = 0$ to $t = 3$?

(A) 3   (B) 6   (C) 9   (D) 18   (E) 27

GO ON TO THE NEXT PAGE.

$$f(x) = \begin{cases} \dfrac{(2x+1)(x-2)}{x-2} & \text{for } x \neq 2 \\ k & \text{for } x = 2 \end{cases}$$

9. Let f be the function defined above. For what value of k is f continuous at $x = 2$?

   (A) 0   (B) 1   (C) 2   (D) 3   (E) 5

10. What is the area of the region in the first quadrant bounded by the graph of $y = e^{x/2}$ and the line $x = 2$?

   (A) $2e - 2$ (B) $2e$ (C) $\dfrac{e}{2} - 1$ (D) $\dfrac{e-1}{2}$ (E) $e - 1$

GO ON TO THE NEXT PAGE.

13. The function f is defined by $f(x) = \begin{cases} 2 & \text{for } x < 3 \\ x - 1 & \text{for } x \geq 3. \end{cases}$ What is the value of $\int_1^5 f(x)\, dx$?

(A) 2   (B) 6   (C) 8   (D) 10   (E) 12

14. If $f(x) = \sqrt{x^2 - 4}$ and $g(x) = 3x - 2$, then the derivative of $f(g(x))$ at $x = 3$ is

(A) $\dfrac{7}{\sqrt{5}}$ (B) $\dfrac{14}{\sqrt{5}}$ (C) $\dfrac{18}{\sqrt{5}}$ (D) $\dfrac{15}{\sqrt{21}}$ (E) $\dfrac{30}{\sqrt{21}}$

GO ON TO THE NEXT PAGE.

16. A particle moves along the x-axis with its position at time t given by $x(t) = (t - a)(t - b)$, where a and b are constants and $a \neq b$. For which of the following values of t is the particle at rest?

(A) $t = ab$

(B) $t = \dfrac{a + b}{2}$

(C) $t = a + b$

(D) $t = 2(a + b)$

(E) $t = a$ and $t = b$

18. $\displaystyle\lim_{h\to 0}\dfrac{\ln(4+h)-\ln(4)}{h}$ is

(A) 0   (B) $\dfrac{1}{4}$ (C) 1   (D) e (E) nonexistent

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19. The function f is defined by $f(x) = \dfrac{x}{x+2}$. What points (x, y) on the graph of f have the property that the line tangent to f at (x, y) has slope $\dfrac{1}{2}$?

(A) $(0,0)$ only

(B) $\left(\dfrac{1}{2}, \dfrac{1}{5}\right)$ only

(C) $(0,0)$ and $(-4, 2)$

(D) $(0,0)$ and $\left(4, \dfrac{2}{3}\right)$

(E) There are no such points.

22. Let f be the function defined by $f(x) = \dfrac{\ln x}{x}$. What is the absolute maximum value of f?

(A) 1

(B) $\dfrac{1}{e}$

(C) 0

(D) $-e$

(E) f does not have an absolute maximum value.

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23. If $P(t)$ is the size of a population at time t, which of the following differential equations describes linear growth in the size of the population?

(A) $\dfrac{dP}{dt} = 200$

(B) $\dfrac{dP}{dt} = 200t$

(C) $\dfrac{dP}{dt} = 100t^2$

(D) $\dfrac{dP}{dt} = 200P$

(E) $\dfrac{dP}{dt} = 100P^2$

GO ON TO THE NEXT PAGE.

25. Which of the following is the solution to the differential equation $\frac{dy}{dx} = 2\sin x$ with the initial condition $y(\pi) = 1$?

(A) $y = 2\cos x + 3$

(B) $y = 2\cos x - 1$

(C) $y = -2\cos x + 3$

(D) $y = -2\cos x + 1$

(E) $y = -2\cos x - 1$